A Theorist's View on

Practical Preferential Voting Rules

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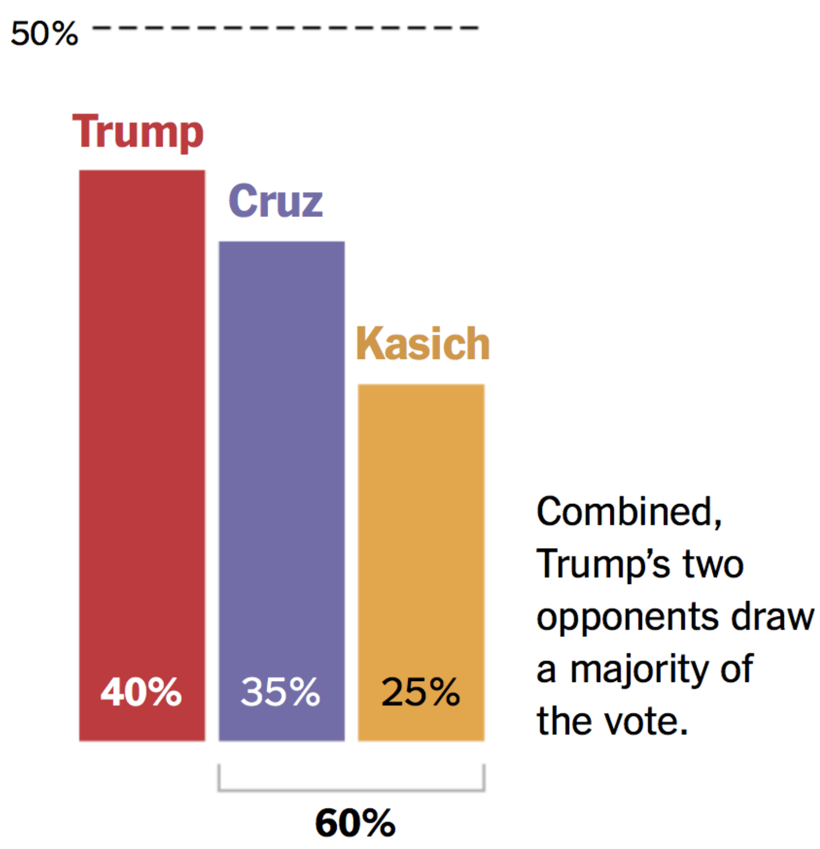
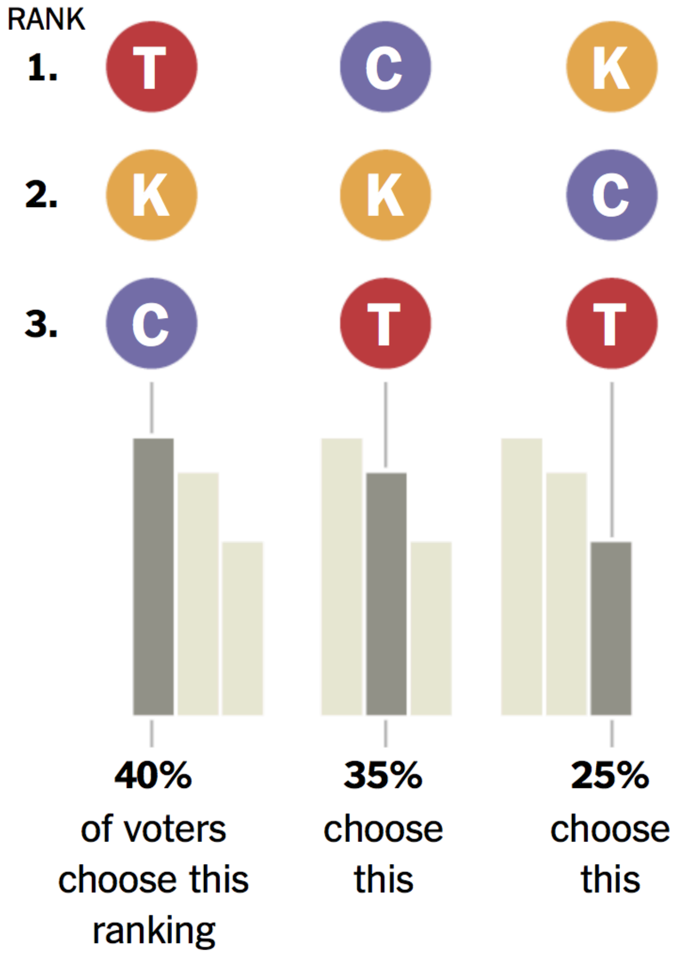
DSS
Decision Sciences & Systems

How to select a single alternative based on the preferences of multiple voters?

Why Preferential Voting?

How majority rule might have stopped Donald Trump (E. Maskin and A. Sen, New York Times, April 2016)





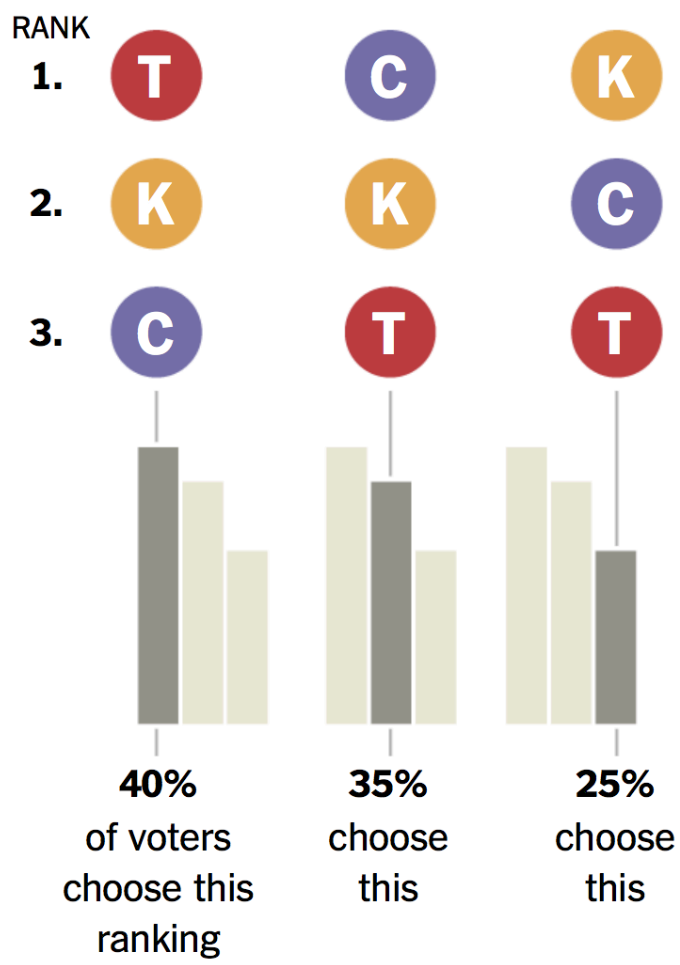
Eric S. Maskin

Amartya K. Sen

Why Preferential Voting?

How majority rule might have stopped Donald Trump (E. Maskin and A. Sen, New York Times, April 2016)

- T is the worst choice according to a majority of voters.
- If the preferences of all voters are reversed, T still wins.
- T loses all pairwise majority comparisons.
- K wins all majority comparisons (Condorcet winner).
- In a poll conducted among 22 leading social choice theorists at *Chateau du Baffy* (France) in 2010, Plurality received no support at all (among 18 voting rules).



Voting Rules Matter

5	4	3	2
a	*e*	*d*	*b*
c	*b*	*c*	*d*
b	*c*	*b*	*e*
d	*d*	*e*	*c*
e	*a*	*a*	*a*

- **Plurality** → *a*
  used in US, Mexico, South Korea, …
- **Borda**
  used in Slovenia, at Harvard University, ESC, …
- **Schulze**
  used by Pirate Party, Wikipedia, Debian, …
- **Instant-runoff**
  used in Canada, UK, Hollywood (Academy Awards), …
- **Plurality with runoff**
  used in France, Brazil, Russia, …

Voting Rules Matter

	5	4	3	2
4	*a*	*e*	*d*	*b*
3	*c*	*b*	*c*	*d*
2	*b*	*c*	*b*	*e*
1	*d*	*d*	*e*	*c*
0	*e*	*a*	*a*	*a*

a: 5x4=20
(b): 5x2+4x3+3x2+2x4=36
c: 5x3+4x2+3x3+2=34
d: 5+4+3x4+2x3=27
e: 4x4+3+2x2=23

- **Plurality**
  used in US, Mexico, South Korea, … → a
- **Borda**
  used in Slovenia, at Harvard University, ESC, … → b
- **Schulze**
  used by Pirate Party, Wikipedia, Debian, …
- **Instant-runoff**
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- **Plurality with runoff**
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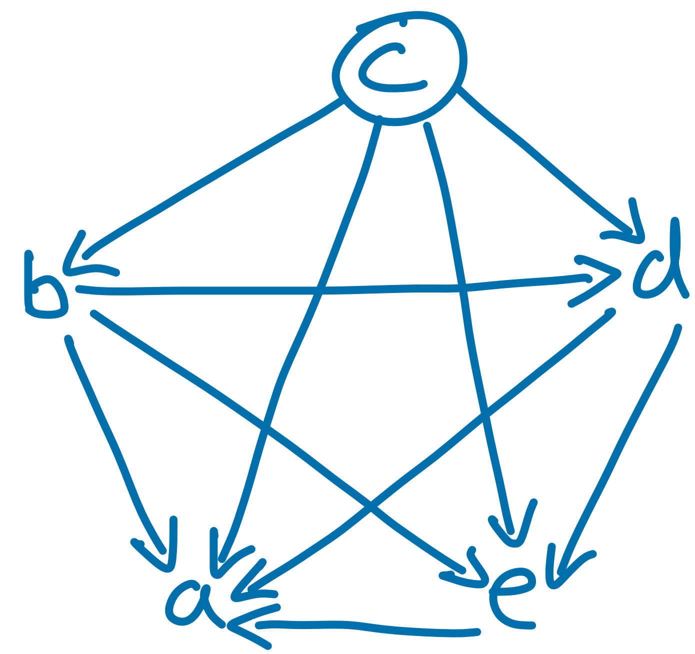
a	*e*	*d*	*b*
c	*b*	*c*	*d*
b	*c*	*b*	*e*
d	*d*	*e*	*c*
e	*a*	*a*	*a*



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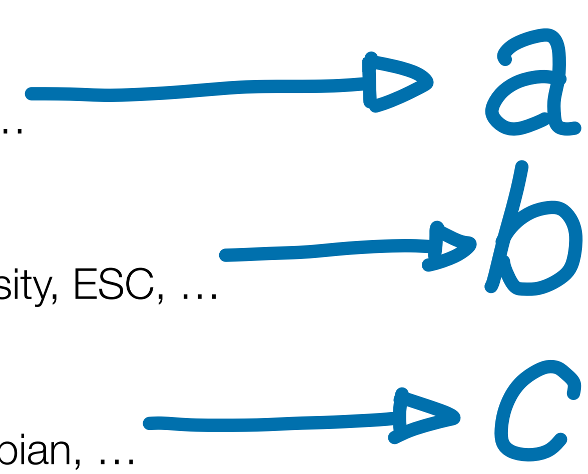

Voting Rules Matter

5	4	3	2
a	[illegible]	*d*	[illegible]
[illegible]	[illegible]	[illegible]	*d*
[illegible]	[illegible]	[illegible]	[illegible]
d	*d*	[illegible]	[illegible]
[illegible]	*a*	*a*	*a*

- **Plurality** → *a*
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Voting Rules Matter

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a	*e*	*d*	*b*
c	*b*	*c*	*d*
b	*c*	*b*	*e*
d	*d*	*e*	*c*
e	*a*	*a*	*a*

- **Plurality** → *a*
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  used in France, Brazil, Russia, …

	D. Felsenthal (2018): On Paradoxes Afflicting Voting Procedures	Plurality	Borda	Schulze	IRV	Runoff
single profile	Condorcet winner paradox	⚠️	⚠️	—	⚠️	⚠️
	Absolute majority paradox	—	⚠️	—	—	—
	Condorcet loser paradox	⚠️	—	—	—	—
	Absolute loser paradox	⚠️	—	—	—	—
	Pareto paradox	—	—	—	—	—
multi profile	Additional support paradox	—	—	—	⚠️	⚠️
	Reinforcement paradox	—	—	⚠️	⚠️	⚠️
	No-Show paradox	—	—	⚠️	⚠️	⚠️
	Twin paradox	—	—	⚠️	⚠️	⚠️
	Subset choice paradox	⚠️	⚠️	⚠️	⚠️	⚠️
	Preference inversion paradox	⚠️	—	—	⚠️	⚠️

Three Desiderata

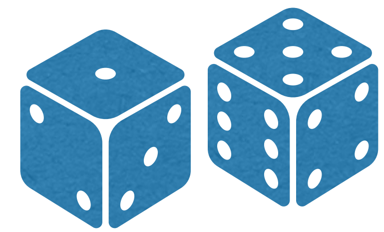
Voting rules…

- should not require strict, complete, or transitive preferences
  - Insistence on strict rankings impedes preferential rules.
  - Pairwise (aka "C2") rules allow great input flexibility.
- should satisfy desirable properties
  - e.g., Pareto-optimality, participation, reinforcement, …
  - even when preferences fail to be strict, complete, or transitive
- should be simple and easy to compute
  - need not necessarily be easily comprehensible by general public
  - in particular, should allow for easy verification of result

Condorcet Winners

- Whenever Condorcet winners exist, all of these desiderata (plus strategyproofness) can be achieved by selecting the Condorcet winner.
- In a vast majority of cases, Condorcet winners do exist!
  - Feld and Grofman (1992) analyze election data from 36 real-world elections, all of which admitted a Condorcet winner.
  - Summarizing 37 empirical studies from 1955 to 2009, Gehrlein and Lepelley (2011) conclude that "*there is a possibility that Condorcet's Paradox might be observed, but that it probably is not a widespread phenomenon.*"
  - For 4 alternatives, the probability of a Condorcet winner is *at least* 82% under the (unrealistic) impartial culture assumption.
- For few alternatives, *any* Condorcet extension will do.

Maximal Lotteries



- Randomized voting rule proposed by Kreweras (1965) and Fishburn (1984)
  - rediscovered by Laffond et al. (1993), Felsenthal and Machover (1992), Fisher and Ryan (1995), Rivest and Shen (2010)
  - variants known as bipartisan set, essential set, and *scrutin de Condorcet randomisé*
- Returns lotteries that are preferred to any other lottery by an *expected* majority of voters

Germain Kreweras

Peter C. Fishburn

Example

5	**4**	**3**	**2**
a	*e*	*d*	*b*
c	*b*	*c*	*d*
b	*c*	*b*	*e*
d	*d*	*e*	*c*
e	*a*	*a*	*a*

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}^T \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & -4 & -4 & -4 & -4 \\ 4 & 0 & -2 & 8 & 6 \\ 4 & 2 & 0 & 4 & 2 \\ 4 & -8 & -4 & 0 & 6 \\ 4 & -6 & -2 & -6 & 0 \end{pmatrix} \end{array} = (4 \;\; 2 \;\; 0 \;\; 4 \;\; 2) \geq 0$$

- Let $M_{x,y} = |\{i : x \succcurlyeq_i y\}| - |\{i : y \succcurlyeq_i x\}|$.
- A lottery p is maximal if $p^T M \geq \mathbf{0}$.
- p is degenerate if and only if there is a (weak) Condorcet winner.
- In contrast to Condorcet winners, maximal lotteries always exist.

Example

5	4	3	2
a	*e*	*d*	*b*
c	*b*	*c*	*d*
b	*d*	*b*	*e*
d	*c*	*e*	*c*
e	*a*	*a*	*a*

$$\begin{pmatrix} 0 \\ 2/7 \\ 4/7 \\ 1/7 \\ 0 \end{pmatrix}^T \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{pmatrix} 0 & -4 & -4 & -4 & -4 \\ 4 & 0 & -2 & 8 & 6 \\ 4 & 2 & 0 & -4 & 2 \\ 4 & -8 & 4 & 0 & 6 \\ 4 & -6 & -2 & -6 & 0 \end{pmatrix} \end{array} = (4\ 0\ 0\ 0\ 26/7) \geq 0$$

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	Absolute majority paradox	—	⚠	—	—	—	—
	Condorcet loser paradox	⚠	—	—	—	—	—
	Absolute loser paradox	⚠	—	—	—	—	—
	Pareto paradox	—	—	—	—	—	—
multi profile	Additional support paradox	—	—	—	⚠	⚠	(—)
	Reinforcement paradox	—	—	⚠	⚠	⚠	—
	No-Show paradox	—	—	⚠	⚠	⚠	—
	Twin paradox	—	—	⚠	⚠	⚠	—
	Subset choice paradox	⚠	⚠	⚠	⚠	⚠	—
	Preference inversion paradox	⚠	—	—	⚠	⚠	—

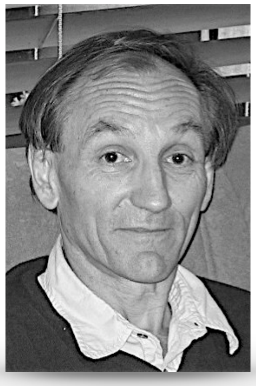
Turning Impossibilities into Characterizations of Maximal Lotteries


Kenneth J. Arrow

- Arrow's impossibility (Arrow, 1951)
  - Brandl and B., Working paper


H. Peyton Young

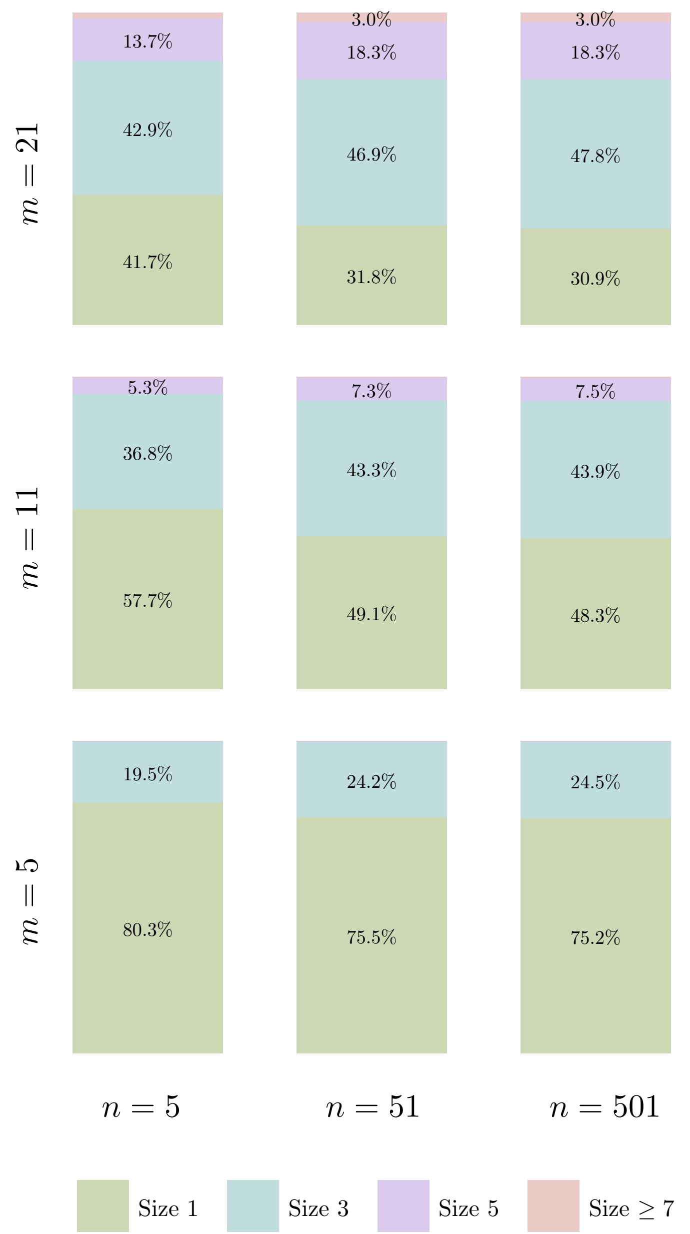
- Reinforcement impossibility (Young & Levenglick, 1978)
  - Brandl, B., and Seedig, Econometrica (2016)


Hervé Moulin

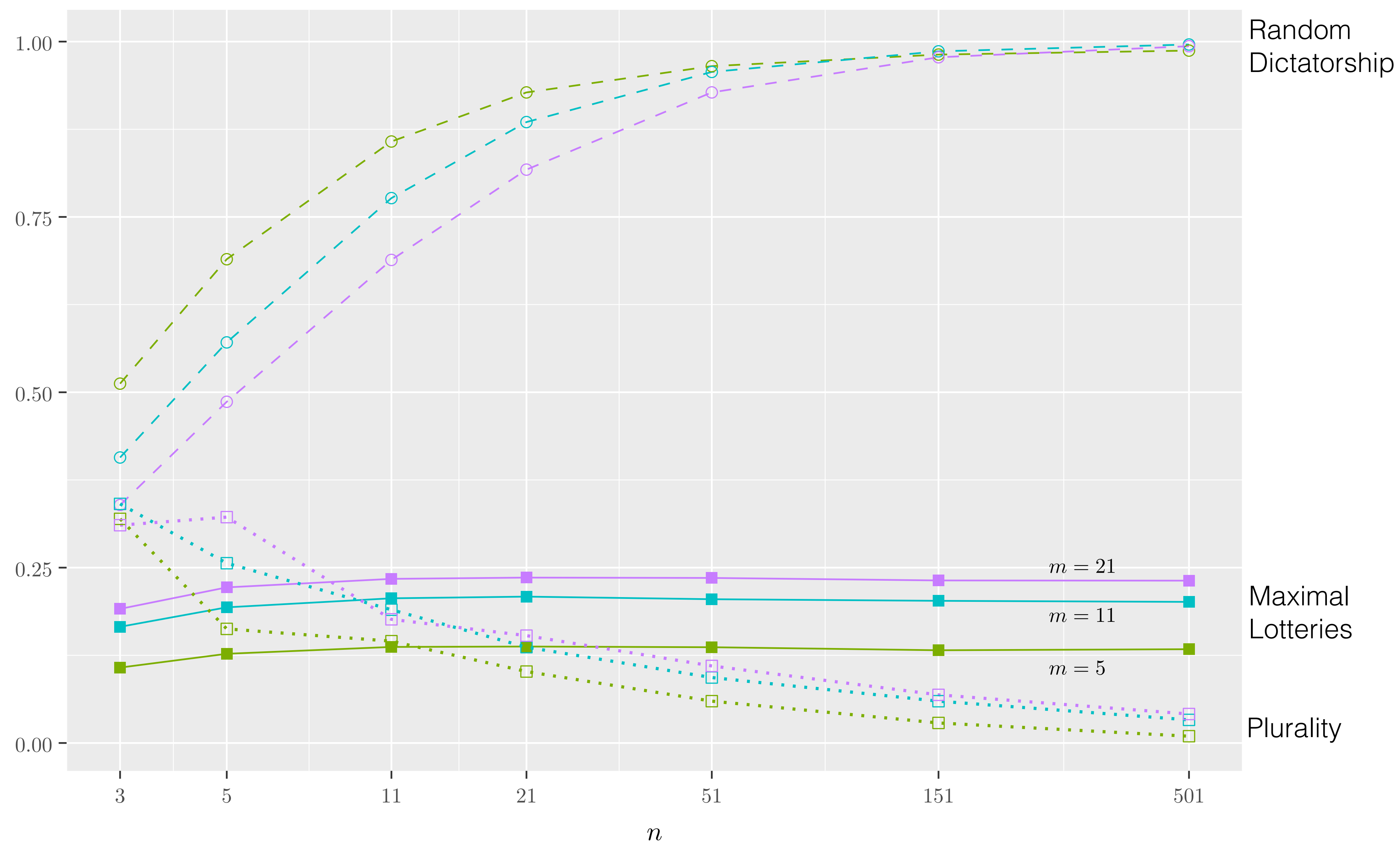
- No-show paradox (Moulin, 1988)
  - Brandl, B., and Hofbauer, GEB (2018)

Degree of Randomization

Maximal Lottery Support Size (IAC)



Shannon Entropy (IAC)

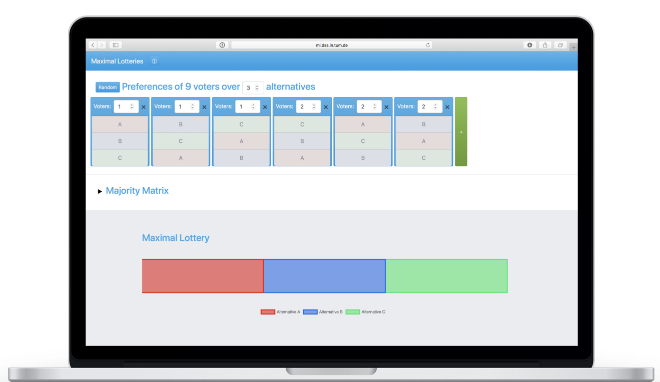


Challenges

- Flexible and expressive specification of preferences
- Educate users about randomization
- Verifiable randomization

[the maximal lotteries system] is not only theoretically interesting and optimal, but simple to use in practice; it is probably easier to implement than, say, IRV. We feel that it can be recommended for practical use.

Rivest and Shen (2010)



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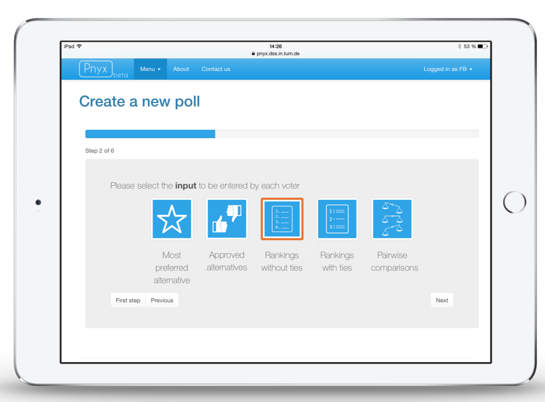

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References

- Aziz, Brandl, B., and Brill. *On the tradeoff between efficiency and strategyproofness*. **Games and Economic Behavior**, 110:1-18, 2018.
- Brandl and B. Arrovian aggregation of convex preferences. Working paper.
- Brandl, B., and Hofbauer. *Welfare maximization entices participation*. **Games and Economic Behavior**, forthcoming.
- Brandl, B., and Seedig. *Consistent probabilistic social choice*. **Econometrica**, 84(5):1839-1880, 2016.
- Brandl, B., and Stricker. An analytical and experimental comparison of maximal lottery schemes. **IJCAI 2018**.